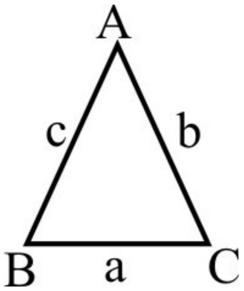
TRIANGLE

Definition : A closed figure formed by three sides.



- A, B and C are vertices and a, b, c are sides of triangle ABC.
- Sum of two sides of a triangle is always greater than third side of that triangle.

(i) a+b>c (ii) b+c>a (iii) c+a>b

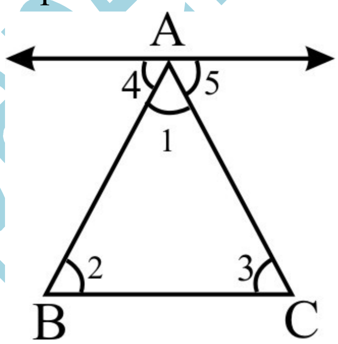
- Difference of two sides of a triangle is always smaller than the third side of that triangle.

(i) $|a-b|<c$ (ii) $|b-c|<a$ (iii) $|c-a|<b$

- **Theorem** : sum of all interior angle of a triangle is 180°.

Proof :

Construction : Drawing a line parallel to BC through point A.



$\angle 2 = \angle 4 ----(1)$ (alternate angle)

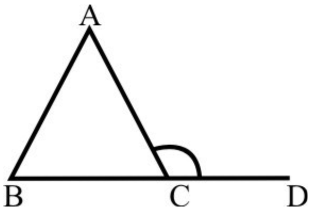
$\angle 3 = \angle 5 ----(2)$ (alternate angle)

$\Rightarrow \angle 1 + \angle 4 + \angle 5 = 180^\circ$ (angles on staright line) $---(3)$

Hence from (i) ,(ii) & (iii)

$\angle 1 + \angle 2 + \angle 3 = 180^\circ$

Theorem : The exterior angle of a triangle is equal sum of two opposite interior angles.



$\angle ACD = \angle A + \angle B$

Proof :

$\angle A + \angle B + \angle C = 180° ----(1)$

$\angle C + \angle ACD = 180° -----(2)$ (linear pair)

From (1) & (2)

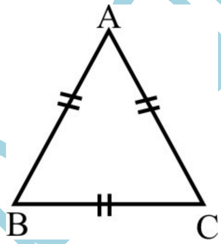
$\Rightarrow \angle A + \angle B + \angle C = \angle C + \angle ACD$

$\Rightarrow \angle A + \angle B = \angle ACD$

Types of Triangle :-

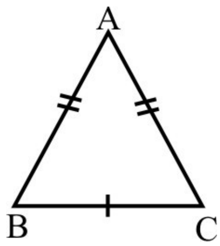
(i) On the basis of sides-

(a) Equilateral Triangle :- A triangle which has all three sides equal is called equilateral triangle.



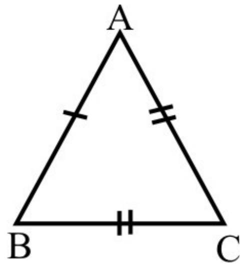
In equilateral triangle $\Rightarrow \angle A = \angle B = \angle C$

(b) Isosceles Triangle :- A triangle which has two equal sides is called isosceles triangles.



[Angles opposite to equal sides are also equal $\angle B = \angle C$]

(c) Scalene Triangle :- A triangle which has three different sides is called scalene triangle.



(ii) On the basis of Angles :-

(a) Acute Angle Triangle :- A triangle which have all angles less than 90° is called acute angle triangle.

- Also, in acute angle triangle $a^2 + b^2 > c^2$

Where c is longest side of triangle.

(b) Right Angle Triangle :- A triangle in which one of angles is 90° is called right angle triangle.

- Also in Right angle triangle.

$a^2 + b^2 = c^2$

Where C is the longest side.

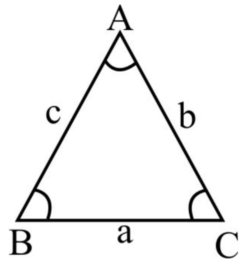
(a) **Obtuse Angle Triangle**- A triangle in which one of angles is obtuse angle is called obtuse angle triangle.

- Also in obtuse angle triangle.

$a^2 + b^2 < c^2$

Where C is the longest side.

- **Relationship of sides to interior angles in a triangles.**
  - The shortest side is always opposite the smallest interior angle.
  - The long side is always opposite the largest interior angle.
  - In ΔABC



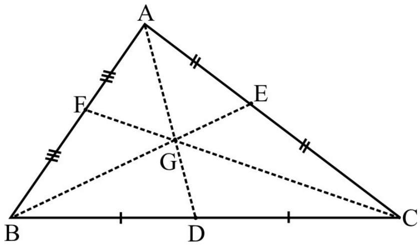
if $a > b > c$

then $\angle A > \angle B > \angle C$

CENTRES OF TRIANGE

1. **Centroid (G)** : Intersection point of three medians is called centroid.

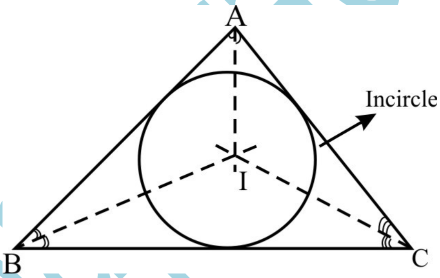
- **Median :**- Line joining from vertex to midpoint of opposite side is called median.



In above triangle, AD, BE and CF are medians and G is the centriod of .

- Centroid divides the median in the ratio 2 : 1.
  AG : GD =BG : GE = CG : GF =2 : 1
- Median divides area of triangle in two equal points.
- Centroid always forms inside in all types of triangle.

2 Incenter (I): Intersection point of angle bisectors of all interior angles in a triangle is called incentre.



ΔABC

I, is the incentre of

Incenter is the center of largest circle. That will fit inside the triangle and touch all three sides.

It is always inside the triangle.

Angles on Incenter:-

$$\angle BIC = 90° + \frac{1}{2}\angle A$$

(i)

$$\angle AIC = 90° + \frac{1}{2}\angle B$$

(ii)

$$\angle AIB = 90° + \frac{1}{2}\angle C$$

(iii)

Proof :

$$\angle A + \angle B + \angle C = 180° -----(1)$$

dividing above equation by 2

$$\Rightarrow \frac{\angle A}{2} + \frac{\angle B}{2} + \frac{\angle C}{2} = 90°$$

$$\Rightarrow \frac{\angle B}{2} + \frac{\angle C}{2} = 90 - \frac{\angle A}{2} -----(2)$$

In ΔBIC

$$\Rightarrow \angle BIC + \frac{\angle B}{2} + \frac{\angle C}{2} = 180 -----(3)$$

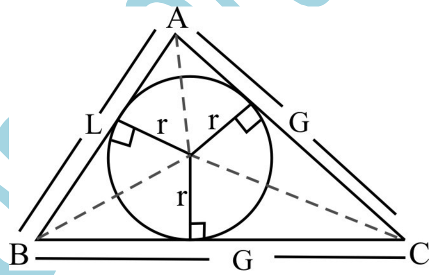
From (2) & (3)

$$\Rightarrow \angle BIC + 90 - \frac{\angle A}{2} = 180$$

$$\Rightarrow \angle BIC = 90 + \frac{\angle A}{2}$$

Inradius (r):

Radius of in circle = $\dfrac{\text{Area of triangle}}{\text{semi perimeter}} = \dfrac{\Delta}{S}$



$$\text{area of } \Delta ABC = \text{Area of } (\Delta BIC + \Delta AIC + \Delta AIB)$$

$$\Rightarrow \text{area of } \Delta ABC = \frac{1}{2} \times BC \times r + \frac{1}{2} \times AC \times r + \frac{1}{2} \times AB \times r$$

$$\Rightarrow \text{area of } \Delta ABC = r\left[\frac{AB + BC + CA}{2}\right]$$

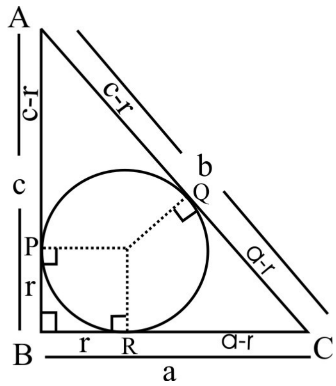
$$\Rightarrow \text{Area of } \Delta ABC = r \times \text{semi.perimeter of } \Delta ABC$$

$$\Rightarrow r = \frac{\text{area of } \Delta ABC}{\text{semi. Perimeter}}$$

<u>Inradius of a Right angle triangle</u>

$$r = \frac{\text{Base} + \text{perpendicular} - \text{Hypotenuse}}{2}$$

Proof:



$\because AP = AQ = C - r$ [$\because$ tangents drawn from external point to circle are equal]

similarly

$CR = CQ = a - r$

Now

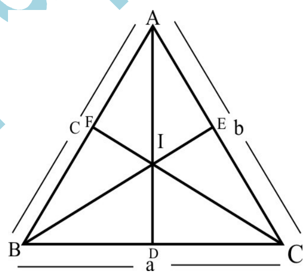
$AC = AQ + CQ$

$\Rightarrow b = c - r + a - r$

$\Rightarrow r = \dfrac{a + c - b}{2}$

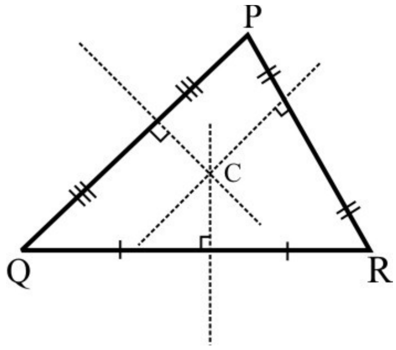
$\Rightarrow r = \dfrac{\text{base} + \text{per.} - \text{hypo}}{2}$

If AD, BE and CF are angle bisector of $\angle A, \angle B$ and $\angle C$ respectively in ΔABC then,

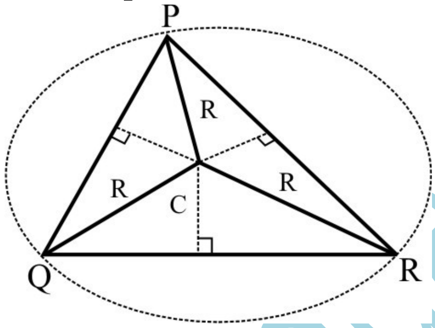


AI	:	ID	BI	: IE	CI	:	IF
$(b+c)$:		a	$(a+c)$:	b	$(a+b)$:		C

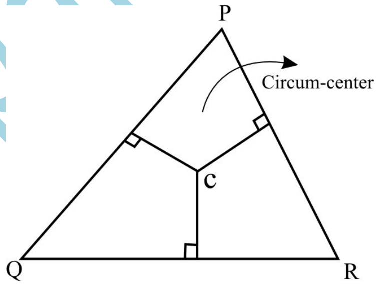
<u>3 Circumcenters</u>:- Intersection point of perpendicular bisectors of three sides of a triangle, is called circumcentre.



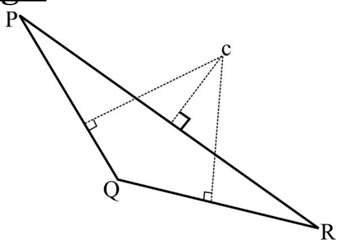
- 'C' is the circumcenter of It is equidistant from all three vertices.



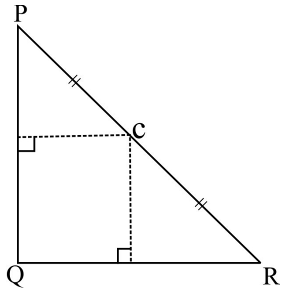
- The distance between circumcenter and one of vertices of triangle is called circumradius.
- If a circle is drawn, taking 'C' as a centre and 'R' as radius. Then it will pass through three vertices of triangle. This circle is called circum circle of triangle.
- **Position of circum center in different triangles.**
  (1) In Acute angle triangle.



- Circumcenter lies inside the triangle.
  (2) **In obtuse-angle triangle**: It lies outside the triangle.



(3) **In Right angle triangle**:- It lies on the midpoint of hypotenuse.



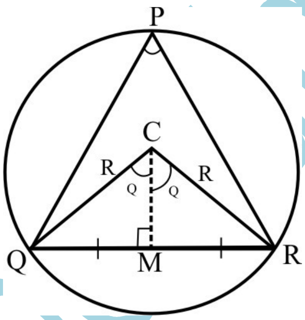
➢ Circum Radius

$$\Rightarrow R = \frac{abc}{4 \times area}$$

Where a, b and C are sides of triangle.

In Right Angle triangle $\Rightarrow R = \frac{Hyptenuse}{2}$

Proof:-



In ΔQMC

$$\sin\theta = \frac{QM}{QC} = \frac{QR/2}{R}$$

$$\Rightarrow \sin\theta = \frac{QR}{2R}$$

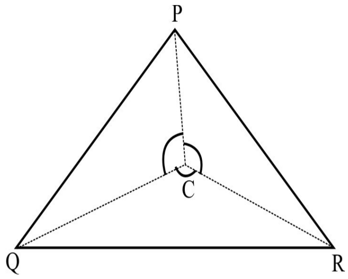
Now

$$\text{area of triangle}(A_t) = \frac{1}{2} \times PQ \times PR \times \text{Sin}\theta$$

$$A_t = \frac{1}{2} \times PQ \times PR \times \frac{QR}{2R}$$

$$\Rightarrow R = \frac{PQ \times QR \times RP}{4 \times A_t}$$

Angles on circum center:-



$\angle QCR = 2\angle P$

$\angle PCR = 2\angle Q$

$\angle PCQ = 2\angle R$

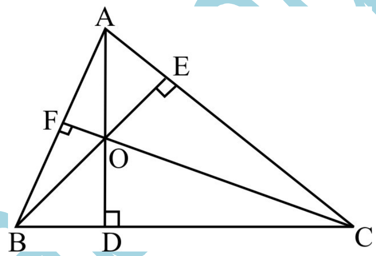
(Proof will be studied in circle)

4 Orthocenter:-

The point where the three attitudes of a triangle intersect is called orthocenter.

Altitude:- An Altitude is a line which passes through a vertex of triangle and meets the opposite side at right angle.

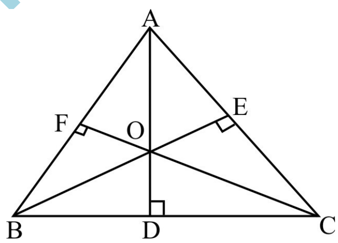
A triangle has three altitudes.



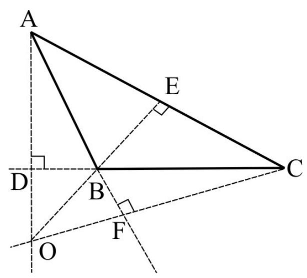
- AD, BE and CF are altitudes and 'O' is orthocenter of ΔABC

- **Position of Orthocenter in different triangle:-**

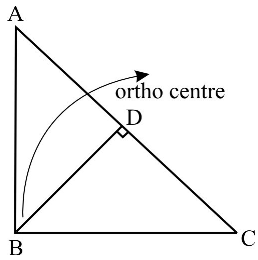
1. **In Acute Angle triangle** : Orthocenter lies inside the triangle.



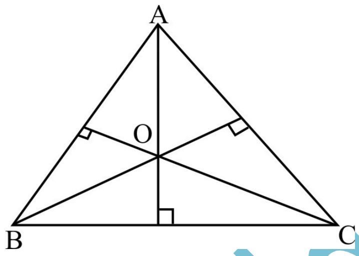
2. **In obtuse Angle Triangle**:- Orthocenter lies outside the triangle.



3 <u>In Right angle Triangle</u> :- Orthocenter lies on the vertex, where 90° angle is formed.



➢ <u>Angles on Orthocentre</u>

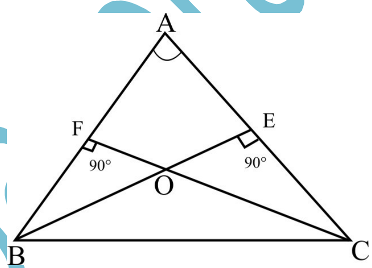


$\angle BOC = 180 - \angle A$

$\angle AOC = 180 - \angle B$

$\angle AOB = 180 - \angle C$

<u>Proof :-</u>



In quadrilateral AF OE

$\angle A + \angle AFO + \angle FOE + \angle AEO = 360°$

$\Rightarrow \angle A + 90° + \angle FOE + 90° = 360°$

$\Rightarrow \angle A + \angle FOE = 180°$

$\Rightarrow \angle FOE = 180 - \angle A$

Q $\angle FOE = \angle BOC$ [vertically opposite angle]

$\therefore \angle BOC = 180 - \angle A$